

**AP Statistics Test
The Normal Distribution
Instructor's Version**

Free Response: 3 questions, 52 points, 30 minutes

- 1. The mean height of adult males is 72 inches, with a standard deviation of 2.5 inches. Consider the heights to follow a normal distribution.**

A. If a man is 71 inches tall, in what percentile does he lie? (6 points)

Answer: He lies in the 34.46th percentile. Area can be found by calculating z-score = -0.4 and using Table A. Using a calculator, normalcdf = .3446.

$$z\text{-score} = (71 - 72)/2.5 = -0.4$$

$$\text{normalcdf}(-E99, 72, 2.5) = .3446$$

B. What percent of men will be between 71 and 75 inches tall? (8 points)

Answer: Roughly 54 percent of men will have heights between 71 and 75 inches. Area can be found by calculating z-scores for 71 inches (from above) and 75 inches, which = 1.2. The student should attempt to find the area under the normal curve between a z-score of -0.4 and 1.2. The area from Table A is .8849 - .3446 = .5403. Using a calculator, normalcdf = .5404.

$$z\text{-score} = (75 - 72)/2.5 = 1.2$$

$$\text{normalcdf}(71, 75, 72, 2.5) = .5404$$

C. How tall must a man be to be in the 98th percentile? (6 points)

Answer: He must be 77.125 inches tall. At the 98th percentile, the area under the curve is .98 with a z-score of 2.05. The student should attempt the equation $Z(\text{std dev}) + \text{mean} = x$ to get $2.05(2.5) + 72 = 77.125$ inches.

$$\text{invnorm}(.98, 72, 2.5) = 77.134$$

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2. The data shown below describe the mean SAT math scores in a recent year for states that have a participation rate of at least 50 percent. Do the data follow a normal distribution? Justify your answer. (16 points)

Note: For convenience, the data have been ordered.

462	465	495	496	496	496	498	499	502	505	506
507	507	509	510	511	512	518	521	522	526	531

Answer: The student must do two things to verify normality or non-normality. An appropriate graph, such as a stemplot or a histogram, should be constructed to see if the graph is approximately bell-shaped and symmetric about the mean. Secondly, the student should compare the count of observations to the 68-95-99.7 rule. Once the student graphs the data, she should comment that there are no distinctly non-normal features such as obvious skewness, gaps, or outliers. This is close enough to the 68-95-99.7 rule, so it appears that the data follow a normal distribution.

mean: 504.27

sample standard deviation: 16.68

+ - 1 standard deviation 16 observations (487.59 - 520.95) 72.7 percent
+ - 2 standard deviations 20 observations (470.91 - 537.63) 90.9 percent
+ - 3 standard deviations 22 observations (454.23 - 554.31) 100 percent

Scoring notes:

- 6 points for a correct graph and stating that it appears "normal"
- 6 points for correct calculation of percentages and comparison to 68-95-99.7 rule
- 4 points for final conclusion

3. Jenny is taking AP Statistics and AP U.S. History. Jenny scored a 95 on an AP Statistics test where the mean score was 84 with a standard deviation = 4.5. She then scored a 90 on an AP U.S. History test with a mean score of 75 and a standard deviation = 8.2. On which test did she perform better relative to others who have taken the test? Justify your answer. (16 points)

Answer: z-score for AP Stat score = $(95-84)/4.5 = 2.44$

z-score for AP U.S. History = $(90-75)/8.2 = 1.82$

Conclusion: Jenny scored better on the AP Statistics test relative to others who have taken the test. This is shown by her higher z-score of 2.44 compared to 1.82 for U.S. History. This shows that her AP Statistics score was further away from the mean score than in U.S. History.

Scoring notes:

- 6 points for correct calculation of AP Statistics z-score
- 6 points for correct calculation of AP U.S. History z-score
- 4 points for correct conclusion with correct reasoning